EXERCISES 1.1.6.

(1) Show that $P \Rightarrow Q$ is the same as $\rceil Q \Rightarrow \rceil P$, which also means that $P \wedge \rceil Q$ is false; a so called *proof by contradiction*. Is $(P \Leftrightarrow Q) \equiv ((P \Rightarrow Q) \wedge (Q \Rightarrow P))$ a reasonable definition? Show that $\rceil (P \wedge Q) = (\rceil P) \vee (\rceil Q)$.

(2) Prove *Morgan's laws* $(\cup X_i)^c = \cap X_i^c$ and $(\cap X_i)^c = \cup X_i^c$.

(3) Show that any partition on a set X is of the form $X/\sim$.

(4) Show that no prime number is the square of a rational number. Hint: Use the prime number factorization theorem. Can you generalize this result?

(5) Prove that convergent sequences in $\mathbb{Q}$ are Cauchy. Show that each real number has at most one positive square root.

(6) Show that the rational numbers are the decimal expansions that are periodic, like $4,567897897897\cdots$ or $0,011111\cdots$. Express $0,32 \equiv 0,32000\cdots$ in a binary expansion.

(7) Show that $f\colon X \to Y$ is bijective iff (meaning, if and only if) it has an *inverse* map, i.e $f^{-1}\colon Y \to X$, and show that the inverse is unique. Show that $\cong$ is an equivalence relation on a set of sets. Prove that $|\mathbb{Z}| = |\mathbb{N}|$ and $|\mathbb{N} \times \mathbb{N}| = |\mathbb{N}|$ and that $|\cup_{n \in \mathbb{N}} X_n| = |\mathbb{N}|$ if all $|X_n| = |\mathbb{N}|$.

(8) Use the Axiom of Choice to show that if one has a surjection $X \to Y$, then $|X| \geq |Y|$.

(9) Show that inverse images respect forming unions and complements. When does inverse images respect single element subsets?