EXERCISES WEEK 1

MARCO MATASSA

1. DE MOIVRE'S FORMULA

The identity below, known as *De Moivre's formula*, shows how complex numbers can be used to derive various identities for trigonometric functions.

Exercise 1.1. Using the basic properties of complex numbers, prove that for any natural number n we have the identity

$$(\cos x + i\sin x)^n = \cos(nx) + i\sin(nx).$$

Hint: use the trigonometric identities

 $\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta, \quad \sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta.$

2. DIFFERENTIAL EQUATIONS

Complex numbers are very useful in solving linear differential equations. Even if we are just interested in real solutions, as is usually the case, they provide a convenient way of handling the computations. This is shown in the next problem.

Exercise 2.1. Consider the differential equation

$$x''(t) + kx(t) = 0,$$

where the prime denotes differentiation with respect to t. Find two real-valued and linearly independent solutions x(t) in the cases k > 0 and k < 0.

Hint: use the exponential $e^{\lambda t}$ by considering $\lambda \in \mathbb{C}$.

OSLOMET - STORBYUNIVERSITETET Email address: marco.matassa@oslomet.no