EXERCISES 2.4.7.

(1) Show that an infinite subset of a compact space has a *limit point*, that is, a point x such that all its neighborhoods will contain points from the subset other than x.

(2) Prove that a function $f\colon X \to Y$ between metric spaces is continuous if and only if $f(x_n) \to f(x)$ for any sequence in X such that $x_n \to x$. Hint: Consider balls with radii $1/n$.

(3) Show that a sequence $E_1 \supset E_2 \supset \cdots$ of non-empty compact subsets of a Hausdorff space has non-empty intersection.