EXERCISES 2.2.6.

(1) Prove that compositions of continuous functions are continuous.

(2) Show that closed subsets of complete metric spaces are complete.

(3) Show that Hausdorffness is a topological invariant.

(4) Prove that the unit circle and $\mathbb{R}$ cannot be homeomorphic, and that none of them are homeomorphic to $\mathbb{R}^2$.

(5) Show that the connected components of $\mathbb{Q} \subset \mathbb{R}$ in the relative topology are the points in $\mathbb{Q}$, and none of these are open.

(6) Prove that the graph of the function $f\colon \langle 0, \infty\rangle \to \mathbb{R}$ given by $f(x) = \sin(1/x)$ together with the origin $(0, 0)$ is a connected subset in the relative topology from $\mathbb{R}^2$, but that it is not arcwise connected.

(7) Show that the continuous real image of a compact connected set is a closed interval, that is, of the form $[a, b]$.

(8) Give examples of continuous functions that are not uniformly continuous, and that are both.